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XVI. Newton's *Binomial Theorem legally demonstrated by Algebra*. By the Rev. William Sewell, A. M. Communicated by Sir Joseph Banks, Bart. K. B. P. R. S.

Read May 12, 1796.

LET  $m$  and  $n$  be any whole positive numbers; and  $1 + \overline{x^n}$  a binomial to be expanded into a series, as  $1 + Ax + Bx^2 + Cx^3 +$ , &c. where  $A, B, C, D$ , &c. are the coefficients to be determined.

Assume  $v^m = \overline{1 + x^n}^{\frac{m}{n}} = 1 + Ax + Bx^2 + Cx^3 + Dx^4 +$ , &c.

And  $z^n = \overline{1 + y^n}^{\frac{m}{n}} = 1 + Ay + By^2 + Cy^3 + Dy^4 +$ , &c.

Then will  $v^n = 1 + x$ , and  $z^n = 1 + y$  . $\therefore$   $v^n - z^n = x - y$ .

And  $v^m - z^m = A \times \overline{x - y} + B \times \overline{x^2 - y^2} + C \times \overline{x^3 - y^3} + D \times \overline{x^4 - y^4} +$ , &c.

Consequently  $\frac{v^m - z^m}{v^n - z^n} = A + B \times \overline{x + y} + C \times \overline{x^2 + xy + y^2} + D \times \overline{x^3 + x^2y + xy^2 + y^3} +$ , &c. Now  $v^m - z^m = \overline{v - z} \times : v^{m-1} + v^{m-2}z + v^{m-3}z^2 +$ , &c.... $z^{m-1}$ . Also  $v^n - z^n = \overline{v - z} \times : v^{n-1} + v^{n-2}z + v^{n-3}z^2 +$ , &c.... $z^{n-1}$ . Therefore  $\frac{v^m - z^m}{v^n - z^n}$  reduces to, and becomes  $= \frac{v^{m-1} + v^{m-2}z + v^{m-3}z^2 + \text{&c....}z^{m-1}}{v^{n-1} + v^{n-2}z + v^{n-3}z^2 + \text{&c....}z^{n-1}}$   
 $= A + B \times \overline{x + y} + C \times \overline{x^2 + xy + y^2} + D \times \overline{x^3 + x^2y + xy^2 + y^3} +$ , &c.

The law is manifest; and it is likewise evident that the

numerator and denominator of the fraction, respectively terminate in  $m$  and  $n$  terms. Suppose then  $x=y$ ; then will  $v=z$ ; and our equation will become  $\frac{mv^{m-1}}{nv^{n-1}}$ , or  $\frac{mv^{m-n}}{n} = A + 2Bx + 3Cx^2 + 4Dx^3 +$ , &c.

But  $v^n = 1 + x$ , therefore by multiplying we have  $\frac{mv^m}{n} = A + A + 2Bx + 2B + 3Cx^2 + 3C + 4Dx^3 +$ , &c. Or  $v^m = \overline{1+x}^{\frac{m}{n}} = \frac{nA}{m} + \frac{nA+2nB}{m}x + \frac{2nB+3nC}{m}x^2 + \frac{3nC+4nD}{m}x^3 +$ , &c. Compare this with the assumed series, to which it is similar and equal, and it will be

$$\frac{nA}{m} = 1$$

$$\frac{nA+2nB}{m} = A$$

$$\frac{2nB+3nC}{m} = B,$$

$$\&c. =, \&c.$$

$$\therefore A = \frac{m}{n}; B = \frac{m-nA}{1.2.n}; C = \frac{m-2nB}{1.2.3.n}; \&c.$$

Therefore  $\overline{1+x}^{\frac{m}{n}} = 1 + \frac{m}{n}x + \frac{m \times m-n}{1.2.n^2}x^2 + \frac{m \times m-n \times m-2n}{1.2.3.n^3}x^3 +$ , &c. the law is manifest, and agrees with the common form derived from other principles.

*Sch.* In the above investigation, it is obvious that unless  $m$  be a positive whole number, the numerator abovementioned does not terminate: it still remains, therefore, to shew how to derive the series when  $m$  is a negative whole number. In this case, the expression  $(v^m - z^m)$  assumes this form,  $\frac{1}{v^m} - \frac{1}{z^m}$ , or its equal  $\frac{z^m - v^m}{v^m z^m}$ , which divided by  $v^n - z^n$ , as before, gives

$$\frac{1}{v^m z^m} \times \frac{z^m - v^m}{v^n - z^n} = \frac{1}{v^m z^m} \times \frac{-v-z \times : v^{m-1} + v^{m-2}z + v^{m-3}z^2}{v- \times : v^{n-1} + v^{n-2}z + v^{n-3}z^2 +}, \&c. =$$

$$\frac{-1}{v^n z^n} \times \frac{v^{n-1} + v^{n-2} z + v^{n-3} z^2 + \&c.}{v^{n-1} + v^{n-2} z + v^{n-3} z^2 + \&c.} = (\text{when } v = z) - \frac{mv^{n-1}}{v^{2n} \times nv^{n-1}}$$

$$= -\frac{mv^{-m-n}}{n}, \text{ which is the same as the expression } \left( \frac{mv^{n-n}}{n} \right) \text{ be-}$$

fore derived with only the sign of  $m$  changed. The remainder of the process being the same as before, shews that the series is general, or extends to all cases, regard being had to the signs. Q. E. D.